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Nonlinear Propagation of Ultrasound in Superfluid ^{3}He

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After introducing the physics of sound propagation in normal and superfluid ^{3}He, nonlinear phenomena are discussed. These bear close resemblance to optical effects, including saturation of the absorption, amplitude dependence of the group velocity, pulse break-up, and pulse compression. Preliminary evidence indicates that above an input power threshold the sound pulses propagate in a solitonlike fashion. A naive sine-Gordon model does not explain the observations.

KEY WORDS: Sound; nonlinear; ^{3}He.

1. INTRODUCTION

Ever since the discovery of the superfluid phases of ^{3}He in 1972, ultrasound measurements have provided valuable information about the liquid's response to density perturbations.[2] Three years ago nonlinear effects were observed in these phases which have analogs in optical systems. The nonlinearities include saturation of the absorption, pulse break-up, amplitude-dependent received pulse widths and group velocities, effects depending on pulse area, and solitonlike pulse scattering results.[(2)] A detailed theory explaining these phenomena has yet to be derived. Since this talk is designed for a multidisciplinary audience, a brief introduction to the basic facts about ^{3}He will be presented. Following this, the concepts of first and zerosounds will be introduced along with an elementary discussion of collective modes in the superfluid. Next the experimental observations in both the linear and nonlinear regimes will be described. Finally, theoretical interpretations of the observations will be reviewed.

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[2] For a review see Ref. 1.

^{3}He condenses under one atmosphere at 3 K. It is a spin-1/2 Fermion with a pressure-dependent Fermi temperature of about 1 K. Between the superfluid transition temperature $T_c \sim 2$ mK and roughly 100 mK ^{3}He is a degenerate neutral normal Fermi liquid. Here "normal" means that the low-energy states of the interacting system can be reached by an adiabatic turn-on of the interactions beginning from the noninteracting case. This is to be contrasted with a Bardeen–Cooper–Schrieffer (BCS) superconductor, where T_c is nonanalytic in the interaction strength as that quantity approaches zero. Additionally, one frequently refers to "quasiparticles," by which one means the low-lying localized states obtained by adding or removing a single particle from the interacting system. Figure 1 shows the phase diagram of ^{3}He.[3] All measurements were performed in magnetic fields below 1 kG. The observations made at Cornell were made exclusively

[3] Phase diagram courtesy of D. D. Osheroff, Bell Laboratories, 600 Mountain Avenue, Murray Hill, New Jersey.

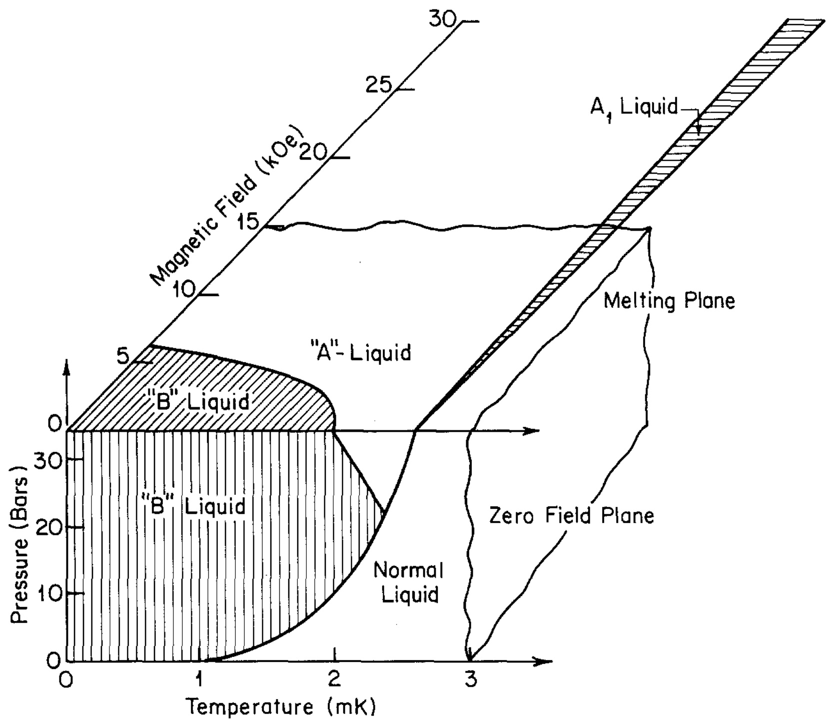

Fig. 1. The low-temperature phase diagram of ^{3}He.

in the B phase; and this talk will confine itself to that fluid, although nonlinear effects have also been seen in the A phase.[(3)]

The normal fluid supports two types of propagating density pertubations. These modes are the result of Landau's Fermi liquid theory.[(4)] This framework is applicable to cases where $\hbar\omega \ll \varepsilon_f$, $\hbar q \ll p_f$, and $T < 100$ mK. Here $\omega(q)$ is the sound frequency (wave vector) and $\varepsilon_f(P_f)$ is the Fermi energy (momentum). Both modes, referred to as zero and first sound, are distortions of the local distribution function:

$$n_p(r, t) = n_p^{\mathrm{EQ}} + \delta n_p(q, \omega)\, e^{i(q \cdot \mathbf{r} - \omega t)}$$

Here the left-hand side represents the probability of finding a quasiparticle at $\mathbf{r}$, t. The first term on the right-hand side is the equilibrium (Fermi) distribution function. δn_p describes the deformation of the local Fermi surface from equilibrium. In response to this "displacement," two types of "restoring forces" are created. In the first, collisions enforce local thermal equilibrium. This requires that the collisions have time to act during one sound wave period, i.e., $\omega\tau \ll 1$. Here τ is a temperature-dependent quasiparticle lifetime of order $\sim 10^{-7}$ sec. This describes first (hydrodynamic) sound, where collisions dominate in the Boltzmann equation for n_p. In an interacting many-particle, a second sort of restoring force is possible: each quasiparticle moves in the self-consistent field generated by all the others. This causes a coherent, cooperative motion, which is upset by collisions. One can neglect collisions provided they induce only a slow change in the distribution function over a wave period: $\omega\tau \gg 1$. This type of density wave is known as zero (collisionless) sound, in analogy with similar types of waves in plasmas. Both types of waves are longitudinally polarized. The usual phase space arguments imply that $\tau \sim 1/T^2$. Then, if one fixes ω in the tens of MHz range (as is done in practice) and lets $T \to 0$, one encounters three regimes of sound propagation: (a) $T > 10$ mK: $\omega\tau \ll 1$ — first sound; (b) $T \sim 8$ mK: $\omega\tau \sim 1$ — large attenuation; (c) $T < 6$ mK: $\omega\tau \gg 1$ — zero sound.

Before discussing zero sound propagation for $T < T_c$, a brief description of the superfluid states will be given.[4] First, consider the ground state of superfluid ^{3}He. Here one has Cooper pairing as in ordinary BCS superconductors, except that the state is spin triplet instead of the $S = 0$ ground state usually encountered in metals. The mechanism of attraction between ^{3}He quasiparticles is not via phonons, as in S-wave superconductors, but the exchange of spin fluctuations is believed to play a major role. In this mechanism, as a spin moves by a point in the fluid it induces a long-lived

[4] For an introduction to superfluid ^{3}He see Ref. 5.

spin polarization cloud there which then attracts a second spin. The superfluid state is characterized by a temperature-dependent gap matrix $\hat{\Delta}(k)$ which varies over momentum ($\mathbf{k}$) space. The meaning of the gap is that energy required to excite a quasiparticle is given by

$$E^2(\hat{k}) = (\varepsilon_{\mathbf{k}} - \varepsilon_f)^2 + \hat{\Delta}\hat{\Delta}^+$$

where ε_k is the quasiparticle kinetic energy. For the B phase, $\hat{\Delta}$ is isotropic in k space and $\hat{\Delta}\hat{\Delta}^+ = \Delta^2\hat{1}$, like a BCS superconductor. More formally, the order parameter may be described by a distribution function g^K which is a 4×4 matrix. The normal fluid, on the other hand, is characterized by a 2×2 matrix distribution function n.

Besides the ground state, excited states of the superfluid may be generated. This is done in practice by passing zero sound through the fluid. The first of these excited states is simply incoherent ionization of the Cooper pairs ("pair breaking") by an external perturbation. This requires $\hbar\omega \geqslant 2\Delta(T)$, the factor 2 arising from the fact that there are two quasiparticles per Cooper pair. Additionally, there exist collective modes of the superfluid. Here the restoring force is created by the quantum mechanical coherence of the pair wave function. It may be mathematically viewed as vibrations of the complex elements of the g^K matrix, corresponding to a ringing or "squashing" of the gap. If one considers Cooper pairs as bound diatomic molecules, these collective modes may also be envisioned as molecular vibrations possessing macroscopic phase coherence. An ordinary gas of diatomic molecules would not support this type of cooperative motion.[5]

Experimentally, these order parameter collective modes appear as maxima in the zero sound attenuation. These may be crudely viewed as arising from the absorption of sound wave energy by a resonantly excited mode. As usual, the energy exchange occurs when the sound frequency and wave vector match that of a collective mode. This can be shown to happen when $\hbar\omega \simeq a(p, H)\,\Delta(T)$. Here $a(p, H)$ is a pressure- and magnetic-field-dependent number on the order of unity. Usually ω is held fixed and T allowed to vary until the resonance condition is satisfied. This leads to plots of sound attenuation $\alpha(T)$ showing maxima at resonant temperatures.

2. EXPERIMENTAL OBSERVATIONS

The experimental arrangement was that of a simple pulse propagation experiment. Radio frequency tone bursts of 100 MHz a few μs long were applied to one of two piezoelectric quartz crystal transducers separated by

[5] For an introduction to collective mode physics in ^{3}He see Ref. 6.

0.318 cm of superfluid ^{3}He. The second transducer served as a receiver, whose electrical response was amplified at room temperature and then further processed.

Figure 2 shows the experimentally determined attenuation as a function of temperature.[7,8] These data were taken at signal levels sufficiently low that no nonlinearities were apparent. Near the low-temperature mode γ, the so-called "real squashing mode," several nonlinear phenomena arose as the transmitted power was increased. The first of these to be observed was absorption saturation shown in Fig. 3. As the transmitted pulse power is increased, the attenuation progressively decreases until the absorption line is completely saturated. The energy density required for this to occur is about 1% of the superfluid condensation energy.

The second nonlinear effect that was observed was pulse break-up. At low-input-power levels one receives a single small broad pulse, in accord with the large dispersion associated with the mode crossing. As the power level is increased, small secondary pulses are observed following the first received pulse. These secondary pulses grow and move faster as the input

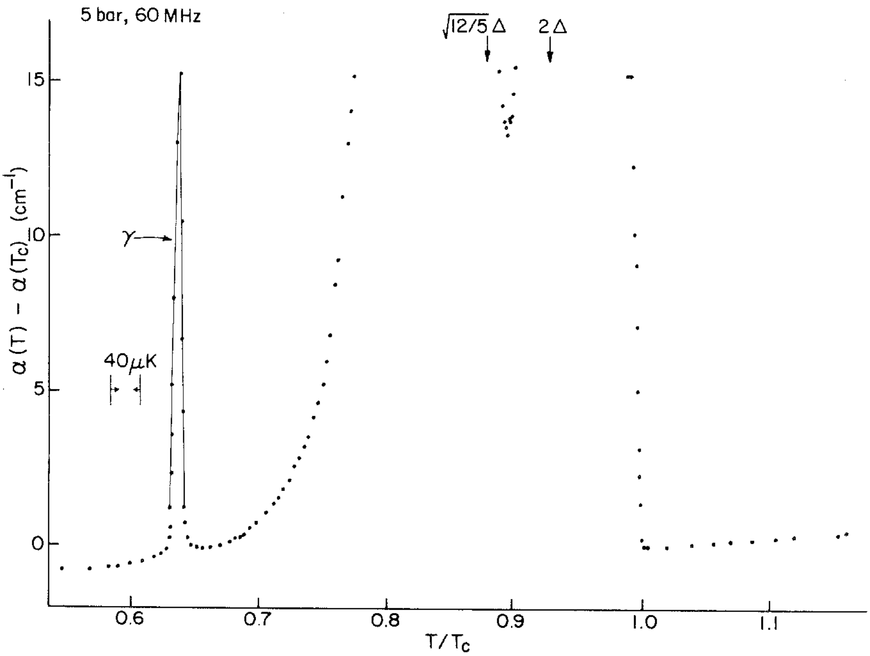

Fig. 2. The temperature-dependent zero sound attentuation coefficient in ^{3}He–B, measured at 5 bars and 60 MHz. The arrow designated 2Δ is the pair breaking peak; $\sqrt{12/5}\Delta$ points to the theoretically determined position of the (imaginary) squashing mode. γ is the mode where the nonlinear effects were studied at Cornell and has been identified as the real squashing mode.

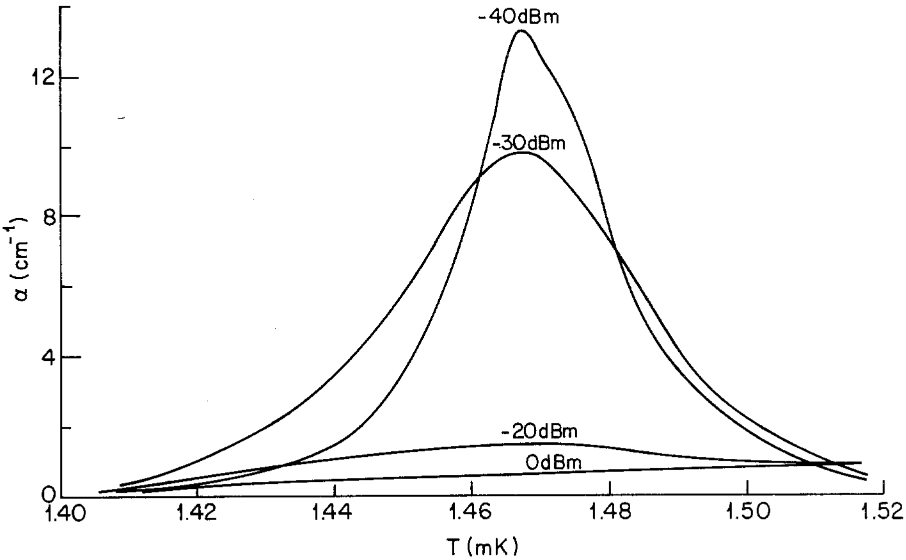

Fig. 3. The power-dependent attentuation coefficient near the γ peak. The power levels labeling each curve are accurate to within 3 dBm. The data was taken at 24 bars with 100 MHz sound.

amplitude is further increased until they overwhelm the original first received pulse. This evolution is shown in Fig. 4. Since the measured bandwidth of the mode was twice the transducer bandwidth or pulse spectral width, hole burning, followed by beating of the spectral wings of the unabsorbed pulse, cannot explain the data. However, the mode width results are still controversial.[8]

Additionally, it was observed that the width and group velocity of the first received pulse were amplitude dependent. These relationships are shown in Fig. 5.

Furthermore, several effects relating to the pulse area (moment) were observed. By area is meant the integral $\int |V(t)|\, dt$, where V is the received signal voltage. Figure 6 illustrates this phenomenon. For constant temperature, the fact that the areal plot of pulses of different duration are coincident eliminates incoherent bleaching as an explanation of the data. For low-input areas the plot obeys Beer's law (linear absorption). Then as the input area is increased there is a sudden change to a regime of constant output pulse area. When the input area is increased to the point where the secondary pulses emerge, the output area increases by several orders of magnitude. It should be noted that the secondaries were of uniform phase, while the first received pulses at low power levels were phase and frequency modulated.

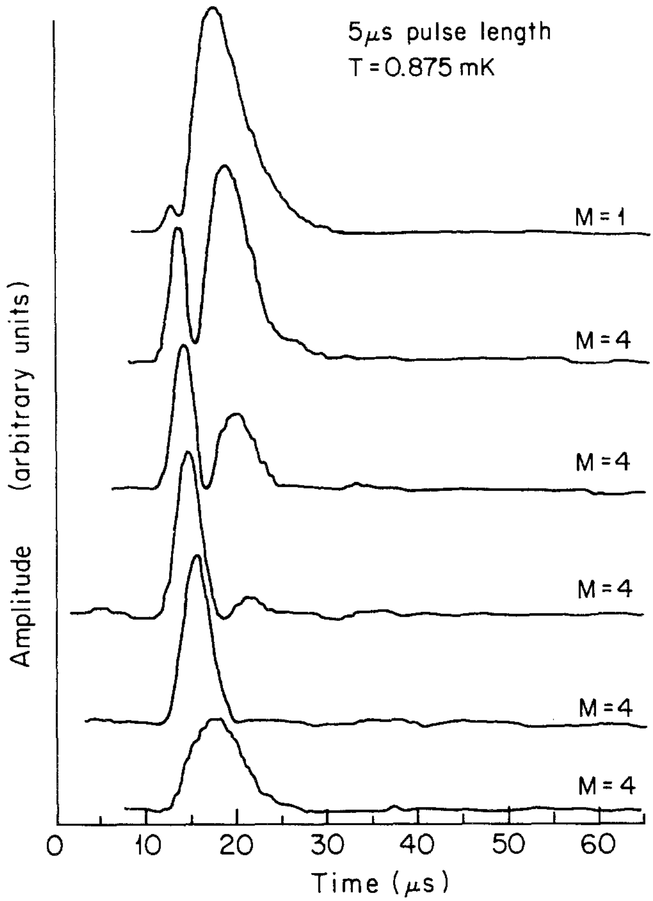

Fig. 4. The evolution of received temporal pulse shape as a function of input power (pulse break-up). M designates a factor by which the curve amplitude should be magnified. Power increases upwards.

At this point evidence seemed to indicate the possibility of solitonlike behavior in this system. To further test this speculative hypothesis pulse scattering experiments were performed. Here both crystals were simultaneously excited and only one acted as a receiver following the front-end amplifier's recovery from saturation. Two different types of results were observed at the receiver. Either the pulses annihilated one another, leading to hash at the receiver, or they superposed. In the normal liquid, superposition occurred everywhere except for cases of amplifier ringing. The same results were obtained in the superfluid far from the mode crossing. These results are in accord with linear theory. In the wings of the line,

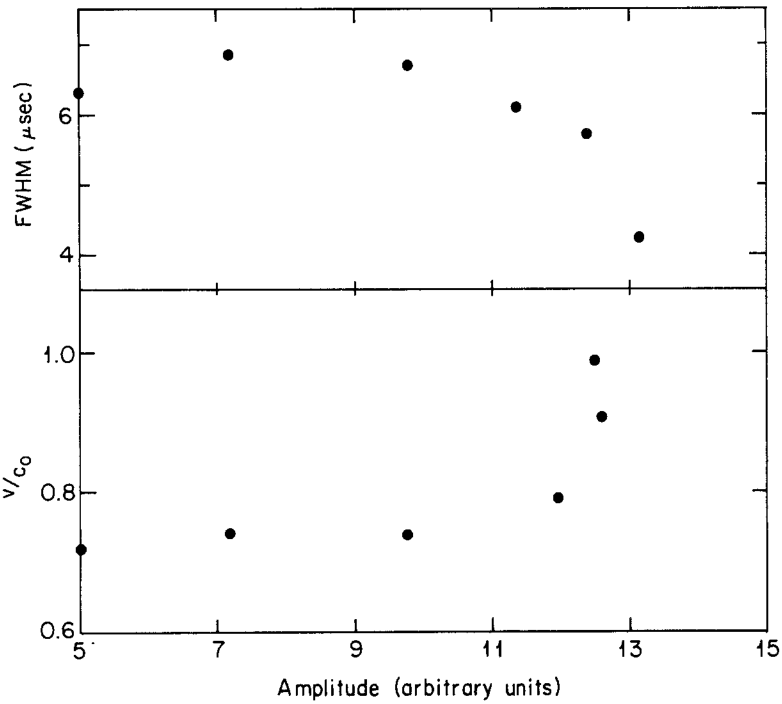

Fig. 5. The amplitude dependent width and group velocity v for the first received pulses. c_0 is the velocity of zero sound just about T_c.

annihilation was observed at all measured powers. On the absorption peak there were three subcases: Low-amplitude (linear) pulses superposed. In the constant-area or multiple-pulse regime, pulses annihilated one another. Finally, the large-area secondary pulses of uniform phase mentioned above superposed. These results are consistent with (but do *not* by themselves imply) solitonlike propagation of the large-area secondaries. The results for the line wings are puzzling in that a linear regime, where one expects superposition to hold, seems to be missing. Perhaps the wings are already partially saturated by powers necessary to obtain reasonable S/N ratios. Moreover, one would have expected the constant-area on-peak results to superpose, based on a soliton picture similar to that used in optical self-induced transparency. Further experiments of this nature are in progress.

3. THEORETICAL INTERPRETATIONS

The first point of view one can take toward these phenomena is a two-level description where the superfluid ground state plays the role of the

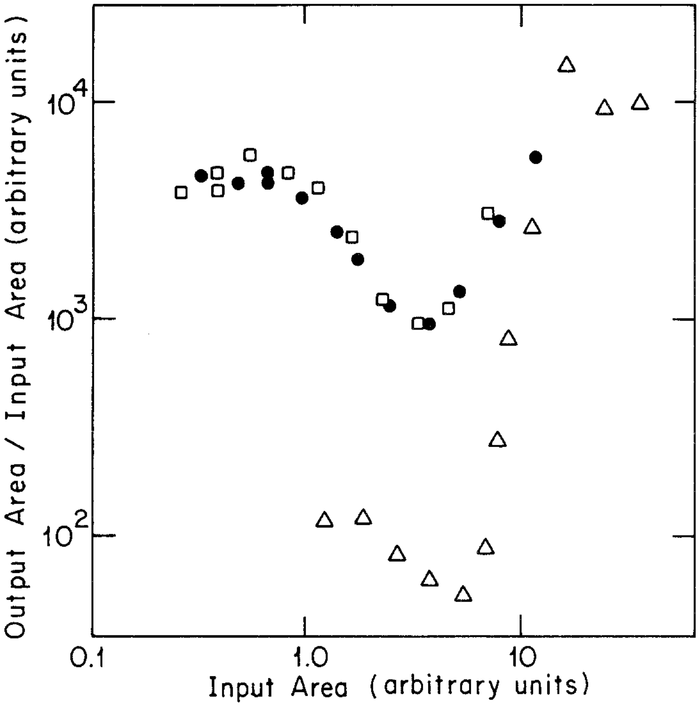

Fig. 6. Total area under the detached signal, divided by the input area, vs. the input area. Data points are $T = 0.984$ mK, 4.6-μsec pulses (△); $T = 0.908$ mK, 1.3 μsec (●); 7-μsec pulses at the same temperature (□).

lower state and the collective mode acts as the upper state. Although the upper state is five-fold degenerate, only one state in the manifold couples to density fluctuations in zero field. The equations resulting from this diatomic molecule analogy are identical to those describing optical self-induced transparency.[9] In particular, one expects an area theorem: $dA/dz = -\alpha \sin A$. Unfortunately, this does *not* agree with the data. Besides the problems already mentioned above, there are three additional difficulties.[10] Firstly, if one plots $A(\text{out})/A(\text{in})$ vs. $A(\text{in})$ for the numerically integrated sine-Gordon equation and for the experiments, the plots are qualitatively different. The theory predicts a sharp rise in $A(\text{out})/A(\text{in})$ for $A(\text{in}) = \pi$, while the experiments show a fall. The theory also has only algebraic variations of $A(\text{out})/A(\text{in})$, while the experimental range is over three orders of magnitude! Secondly, the two-level model predicts $c_0/v - 1 \sim (\tau_F)^n$, where c_0 is the normal fluid zero-sound velocity, v is the measured group velocity, and τ_F is the pulse temporal width. $n = 1(2)$

if the mode linewidth $\gg$ ($\ll$) pulse spectral width. Both $n = 1, 2$ fit the $A(\text{out}) = \text{constant}$ portion of the data, making this test inconclusive. Finally, the self-induced transparency model gives $\tau_F \sim (\text{amplitude})^{-1}$ for $A(\text{out}) = 2\pi$, but the data fit $\tau_F \sim \exp(-\text{const} \times \text{ampl})$ better.

A recent preprint[11] expands the distribution functions g^K and the molecular fields about equilibrium but goes beyond first order in the density δn to third and higher orders. The results are that for the normal liquid the nonlinear effects ought to be much smaller (by three orders of magnitude) than in the superfluid. For ^{3}He–B third harmonic generation is predicted for $\delta E_{\text{sound}} \sim (1\text{-}5\%)\, E_{\text{pair}}$, which is in the regime of the observed saturation effects.

4. CONCLUSIONS

Collective modes in superfluid ^{3}He have some features in common with optical nonlinear systems. These include absorption saturation, pulse break-up and compression, amplitude dependent velocities, and effects relating to pulse area. The scattering results are complex but are consistent with solitonlike propagation of the large single phase secondary pulses near the resonance. Finally, the naive sine-Gordon model is not a faithful paradigm for the system. More experimental and theoretical work is necessary before a clear understanding of these rich phenomena is reached.

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